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DEMONSTRATION OF A PROPOSITION.

BY P. F. MANGE, ALAMOS, SONORA, MEXICO.

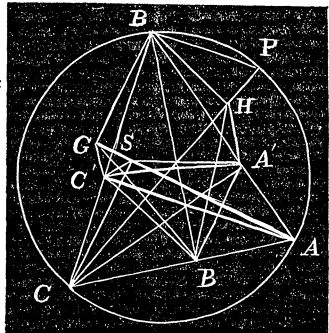
Proposition. The area of any triangle is equal to the radius of the circumscribing circle multiplied by half the perimeter of the triangle formed by joining the feet of the perpendiculars drawn from the angles of the given triangle to the opposite sides.

Let ABC be the given triangle, O the centre of the circumscribed circle, and call its radius R . Also, let $A'B'C'$ be the triangle formed by joining the feet of the perpendiculars.

Join AO , BO and CO ; prolong CO to meet the circumference in P , and also AO to meet a perpendicular from B in G . Draw BH and BG respectively perpendicular to CP and AG ; draw CS also perpendicular to AG at the point S , and join BP , HB' , $A'S$, $B'G$, HA' , $G'C'$, and $C'S$.

We have angle $BPC = BAC$, and right angle $PBC = AA'C$, therefore $PCB = ACA'$. Now as angles BHC , $BA'C$, and $BB'C$ are all right angles, it follows that the points B , H , A' , B' , and C are all in the semicircumference of a circle whose diameter is BC , consequently the arcs BH and $A'B'$ are equal; $\therefore$ their chords are also equal; hence $BH = A'B'$. By the same course of reasoning it may be shown that $BG = B'C'$ and $CS = A'C'$.

Now the area of triangle ABC is evidently equal to the sum of the areas of the triangles BOC , BOA and AOC ; but we know that the area of trian. $BOC = \frac{1}{2}BH.CO = \frac{1}{2}A'B'.R$,
 " " " $BOA = \frac{1}{2}BG.AO = \frac{1}{2}B'C'.R$,
 " " " $AOC = \frac{1}{2}CS.AO = \frac{1}{2}A'C'.R$.



Hence by addition we get
 " " " $ABC = \frac{1}{2}(A'B' + B'C' + A'C')R$. Q. E. D.

SCH. I. Denoting the sides of the triangle ABC by a , b , c , and those of the pedal triangle by a' , b' , c' ; their respective areas by K and K' and the radius of the inscribed circle by r , we have, by the above demonstration,

$$K = \frac{1}{2}(a' + b' + c')R; \text{ but we know that } K = \frac{1}{2}(a + b + c)r;$$

$$\therefore \frac{(a' + b' + c')R}{2} = \frac{(a + b + c)r}{2}; \therefore \frac{a' + b' + c'}{a + b + c} = \frac{r}{R}.$$

SCH. II. Denoting the radius of the circle inscribed in the pedal triangle by r' we have as above $K = \frac{1}{2}(a' + b' + c')R$ and $K' = \frac{1}{2}(a' + b' + c')r'$;

$$\therefore \frac{K'}{K} = \frac{r'}{R}.$$

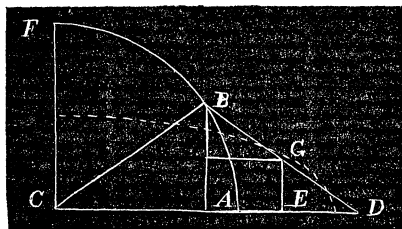
432. By R. J. Adcock.—Show that the quadrant of the ellipse equals

$$a \int_0^1 \left(\frac{1-e^2x^2}{1-x^2} \right)^{\frac{1}{2}} dx = \frac{1}{2}\pi \cos \theta \left[1 + \left(\frac{1}{2} \tan \theta \right)^2 - \frac{1}{3} \left(\frac{1.3}{2.4} \tan^2 \theta \right)^2 + \frac{1}{5} \left(\frac{1.3.5}{2.4.6} \tan^3 \theta \right)^2 - \frac{1}{7} \left(\frac{1.3.5.7}{2.4.6.8} \tan^4 \theta \right)^2 + \&c. \right],$$

where a = semi transverse axis, b semi conjugate, $e^2 = 1 - (b^2 \div a^2)$, $\tan^2 \theta = e^2 \div (1 - e^2)$.

SOLUTION BY THE PROPOSER.

While one foot of the isosceles triangle CBD , $CB = BD = \frac{1}{2}(a+b)$, moves along the axis CD of x , the other foot remaining fixed at the origin C , any p't G in the side BD describes an ellipse of which the semi axes are a and b , $CB = \frac{1}{2}(a+b)$, $DG = b$.



Let the angle $CBD = 2\theta$, then, in the equation $a^2y^2 + b^2x^2 = a^2b^2$,

$$x = \frac{1}{2}(a+b) \cos \theta + \frac{1}{2}(a-b) \cos \theta = a \cos \theta, \quad y = b \sin \theta.$$

$$\sqrt{(dx^2 + dy^2)} = \sqrt{(a^2 \sin^2 \theta + b^2 \cos^2 \theta)} d\theta = \sqrt{(b^2 + c^2 \sin^2 \theta)} d\theta,$$

$$= b \left(1 + \frac{c^2}{b^2} \sin^2 \theta \right)^{\frac{1}{2}} d\theta; \quad c^2 = a^2 - b^2.$$

$$\begin{aligned} \int_0^{\frac{1}{2}\pi} \sqrt{(dx^2 + dy^2)} &= \int_0^{\frac{1}{2}\pi} b \left(1 + \frac{c^2}{b^2} \sin^2 \theta - \frac{1}{2.4} \frac{c^4}{b^4} \sin^4 \theta + \&c. \right) d\theta \\ &= \frac{1}{2}\pi b \left(1 + \left(\frac{1}{2} \right)^2 \frac{c^2}{b^2} \right) - \frac{1}{3} \left(\frac{1.3}{2.4} \frac{c^2}{b^2} \right)^2 + \frac{1}{5} \left(\frac{1.3.5}{2.4.6} \frac{c^3}{b^3} \right)^2 - \&c. \end{aligned}$$

which is the length of the quadrant after supplying the factor a in the second member.

[We may prove that the locus of G is an ellipse as follows:—From the similar triangles ABC , ABD and EGD we have

$$DG : DE :: BG : AE; \quad \text{whence } AE = \frac{DE \times BG}{DG};$$

$$DG : DE :: CB : CA; \quad \text{“} \quad CA = \frac{DE \times CB}{DG}.$$

Putting x , y for coordinates of G and substituting the given values for DG , CB and BG we get

$$x = CA + AE = \frac{\frac{1}{2}(a+b)\sqrt{(b^2-y^2)} + \frac{1}{2}(a-b)\sqrt{(b^2-y^2)}}{b},$$

whence $b^2x^2 + a^2y = a^2b^2$, which is the equation of an ellipse.—Ed.]